

Is a classical description of stable non-BPS D-branes possible ? *

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Abstract

We study the classical geometry produced by a stack of stable (i.e. tachyon free) non-BPS D-branes present in K3 compactifications of type II string theory. This classical representation is derived by solving the equations of motion describing the low-energy dynamics of the supergravity fields which couple to the non-BPS state. Differently from what expected, this configuration displays a singular behaviour: the space-time geometry has a *repulson*-like singularity. This fact suggests that the simplest setting, namely a set of coinciding non-interacting D-branes, is not acceptable. We finally discuss the possible existence of other acceptable configurations corresponding to more complicated bound states of these non-BPS branes.

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1 Introduction

One of the corner-stones of all recent developments in string theory has been the exact microscopic description of D-branes provided by J. Polchinski [1]. D-branes are Ramond-Ramond charged objects defined as hypersurfaces on which open strings can end. From the low-energy point of view, the D-branes instead appear as classical solutions [2] of the supergravity field equations which preserve a fraction of the original set of supercharges, and hence are BPS saturated. The fact that type II supergravity possesses classical BPS solutions was known well before Polchinski's paper (see for instance [3]), but only after the stringy interpretation their fundamental importance has been fully appreciated.

More recently, after a series of papers by A. Sen [4, 5, 6, 7, 8], a lot of attention has been devoted to the study of non-BPS D-branes (for reviews see [9, 10, 11, 12]). The main motivations are the following: *i*) stable non-BPS D-branes are crucial in testing some non-perturbative string dualities without relying on supersymmetry arguments [6, 13, 14, 15, 16]; *ii*) the existence of non-BPS D-branes could be used to define or find duality relations in a non-supersymmetric context [17]; *iii*) non-BPS D-branes hopefully can play a crucial role in describing non-perturbative properties of non-supersymmetric gauge theories, similarly to what happened with the BPS D-branes in connection with supersymmetric Yang-Mills theory.

Despite the big amount of knowledge that has been accumulated, non-BPS D-branes have still to be completely understood. There are two main types of D-branes which do not saturate the BPS bound: those which are unstable due to the

existence of tachyons on their world-volume, and those which are instead stable and free of tachyons. For example, the Dp branes of type IIA with p odd are of the first kind, while the D-particle of type I is of the second kind. In both cases, their microscopic description is fairly well under control, for instance, in terms of world-volume effective actions [18, 19] or boundary states [5, 13, 15, 20]¹, but very little is known about the nature or even the existence of the classical geometry associated to them. Actually, in the case of an unstable non-BPS brane one should first of all specify what is the meaning of a classical solution, but even in the simpler case of the stable non-BPS branes a discussion about the conditions which guarantee the consistency of the classical geometry with the microscopic string description is lacking.

From the effective field theory point of view, the problem of finding the classical solution corresponding to a given brane configuration is always well defined, because it amounts to solve the inhomogeneous field equations of the supergravity theory in the presence of a source term represented by the brane effective action, which has a delta-function singularity at the position of the brane. Equivalently, the same problem can be addressed by solving the homogeneous field equations and then imposing that the solution has the asymptotic behaviour prescribed by the boundary state description of the brane. As we shall see, this procedure clearly shows how the integration constants appearing in the solution are related to the physical parameters of the brane configuration, typically its tension and charges. The existence and the nature of possible singularities of the solution therefore depend on such constants, and the physical requirements that a classical solution should fulfill (for instance, the absence of naked singularities) constrain the acceptable range of their values.

From the microscopic point of view, however, the supergravity action describes only the effective dynamics of the model at low energies, and thus one has to be sure that the constraints imposed on the brane parameters by the existence of a meaningful classical solution of the field equations are compatible with the approximations that lead to such an action. As is well known, this may happen only when the brane tension is very large, *i.e.* $M_p \rightarrow \infty$. The validity of the no-force condition, which allows to construct a superposition of an arbitrary large number of D-branes, is therefore the necessary condition for the consistency of a classical solution.

In this paper we address these questions in general and discuss in particular the

¹For a review of the boundary state formalism and its applications, see [21].

case of the non-BPS D-particle in six dimensions arising from the compactification of the type II string theory on a K3 manifold at the orbifold point. This configuration, which can be easily described using the boundary state formalism, is *stable* because the orbifold projection removes the open string tachyons; moreover, at a particular value of the volume of the compact space it satisfies a *no-force condition* at one loop [22]. This system seems therefore to possess all the required features to produce a non-trivial classical geometry whose leading behavior at large distances has recently been found in [23]. However, differently from what expected, we will give evidences that this does not happen. In particular, we will find that the geometry corresponding to a stack of such non-interacting non-BPS branes displays pathological features which make the configuration unacceptable. This is not in contradiction with the result of [22]. In fact, one-loop calculations can extend their validity to the supergravity regime only when some preserved supersymmetries cancel higher order corrections. In the case of a non-BPS configuration clearly this is not guaranteed and our result is indeed an evidence that the no-force found at first order is lost at two-loop level. It is interesting to observe that the singularities we encounter in our solution manifest themselves as divergences in the metric tensor which make the gravitational force to become “repulsive” at small distances. This kind of singularities are known in the literature as *repulsons* [24], and have been recently considered in string theory in [25] where a mechanism for their resolution has been proposed. It would be interesting to investigate whether a similar mechanism can be applied also in our case.

The content of this paper is the following: in Section 2 we consider the unstable non-BPS D-branes of type II in ten dimensions, and discuss the limits and the validity of the corresponding classical geometry. In Section 3 we compactify the type II string on the orbifold T_4/\mathbf{Z}_2 and write the six-dimensional low-energy effective action which describes the model in the field theory limit. Even though it is known that this action is that of the (1,1) supergravity coupled to 20 $U(1)$ vector multiplets, for our purposes we find more convenient to recover its explicit form using the S-duality which relates our model to the heterotic theory compactified on T^4 . In Section 4, by exploiting the precise knowledge of the action obtained with the duality map, and of the couplings between the fields and the D-brane given by the boundary state, we write down the equations of motion and solve them iteratively in the effective open string coupling gN . Contrarily to what expected, the perturbative series may be resummed to obtain the exact solution with the asymptotic behaviour described by the boundary state. As already stressed, the

solution presents pathologies which signal the impossibility of having a macroscopic stable configuration of N coincident non-BPS D-particles. It may be possible that more complicated bound states made up of stable non-BPS D-branes can resolve these singularities and lead to a consistent classical solution. This issue is discussed in Section 5, where following [26], we solve the system of the homogeneous field equations in full generality and comment on possible different settings in which a macroscopic solution can exist and the singularity be resolved. Finally, in Appendix A and B we present some technical details and explicit calculations.

2 Non-BPS D-branes in type II theories

By now it is well-known that type II string theories in ten dimensions possess non-BPS Dp -branes (with p odd in IIA and p even in IIB)² which are unstable due to the existence of open string tachyons on their world-volumes. If we give a vanishing v.e.v. to such tachyons, these non-BPS D-branes are easily described by a boundary state $|Dp\rangle$ which has only a NS-NS component

$$|Dp\rangle = \sqrt{2} |Bp\rangle_{\text{NS-NS}} \quad (2.1)$$

where $|Bp\rangle_{\text{NS-NS}}$ is the GSO projected boundary state in the NS-NS sector whose explicit expression can be found for example in [21]. The factor of $\sqrt{2}$ in (2.1) is required by the open-closed string consistency and implies that a non-BPS Dp -brane of type IIA (or B) is heavier than the corresponding BPS Dp -brane of type IIB (or A). In fact, from (2.1) one can see that the tension is

$$\widehat{M}_p = \sqrt{2} \frac{T_p}{\kappa_{10} g} \quad (2.2)$$

where $T_p = \sqrt{\pi} (2\pi\sqrt{\alpha'})^{3-p}$ is the factor that appears in the normalization of the boundary state, $\kappa_{10} = 8\pi^{7/2}\alpha'^2$ is the gravitational coupling constant in ten dimensions, and g is the string coupling constant.

By projecting $|Dp\rangle$ onto perturbative closed string states, one can easily see that the only massless bulk fields emitted by the non-BPS D-branes are the graviton $G_{\mu\nu}$ and the dilaton φ , and that their couplings are described by the DBI action (in the

²Throughout this section, we always take $p < 7$.

Einstein frame)³

$$S_{\text{boundary}} = -\widehat{M}_p \int d^{p+1}x \, e^{\frac{p-3}{4}\varphi} \sqrt{-\det G_{\alpha\beta}} \quad . \quad (2.3)$$

The graviton and the dilaton can in principle propagate in the entire ten dimensional space-time where their dynamics is governed by the following bulk action (again in the Einstein frame)

$$S_{\text{bulk}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \, \sqrt{-\det G} \left(\mathcal{R}(G) - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \right) \quad (2.4)$$

which is a consistent truncation of the type IIA (or B) supergravity action containing only those fields emitted by the non-BPS D-brane.

Following the procedure described in [27, 28] and using the explicit form of the boundary state (2.1), one can find the metric and dilaton profiles at large distances from the brane. These turn out to be given by

$$\begin{aligned} G_{\alpha\beta} &\simeq \left(1 + \frac{p-7}{8} \frac{\widehat{Q}_p}{r^{7-p}} + \dots \right) \eta_{\alpha\beta} \\ G_{ij} &\simeq \left(1 + \frac{p+1}{8} \frac{\widehat{Q}_p}{r^{7-p}} + \dots \right) \delta_{ij} \\ \varphi &\simeq \frac{3-p}{4} \frac{\widehat{Q}_p}{r^{7-p}} + \dots \end{aligned} \quad (2.5)$$

where r is the distance in transverse space and

$$\widehat{Q}_p = \frac{2 \widehat{M}_p \kappa_{10}^2 g^2}{(7-p) \Omega_{8-p}} \quad (2.6)$$

with $\Omega_q = 2\pi^{\frac{1}{2}(q+1)} / \Gamma(\frac{1}{2}(q+1))$ being the area of a unit q -sphere. We remark that the expressions (2.5) are the same as those of the usual BPS Dp -branes in the Einstein frame (except for the different value of the tension appearing in \widehat{Q}_p). It is also worth pointing out that the terms in (2.5) proportional to \widehat{Q}_p can be obtained by evaluating the 1-point diagrams for the graviton and the dilaton (see Figure 1) in which the couplings with the brane are read from the boundary action (2.3).

At this point it is natural to ask whether these non-BPS D-branes can yield a non-trivial classical geometry in ten dimensions, just like the BPS D-branes do. In other words, one can ask whether the metric in (2.5) can be interpreted not only

³ We label the world-volume directions of the brane with indices $\alpha, \beta, \dots = 0, \dots, p$ and the transverse directions by indices $i, j, \dots = p+1, \dots, 9$.

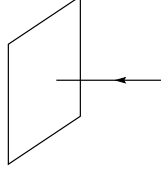


Figure 1: The leading contribution to the one-point function of a bulk field expressed in a diagrammatic way.

as a small deformation of the flat Minkowski space-time due to the emission of a graviton, but also as the asymptotic behavior of a non-trivial space-time geometry. One way to answer this question is to compute higher order terms in \hat{Q}_p , both for the metric and the dilaton profiles, and eventually re-sum the perturbative series. Despite its conceptual simplicity, this procedure clearly requires calculations which become more and more daunting as one proceeds in the perturbative expansion. However, there is another (and more efficient) way to answer the question, namely one can write the field equations of the metric and the dilaton that follow from the bulk action (2.4), and then look for a solution with the asymptotic behavior (2.5). In our case these equations are simply

$$\partial_\mu \left(\sqrt{-\det G} G^{\mu\nu} \partial_\nu \varphi \right) = 0 \quad (2.7)$$

for the dilaton, and

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} \mathcal{R} - \frac{1}{2} \left(\partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} G_{\mu\nu} \partial_\rho \varphi \partial^\rho \varphi \right) = 0 \quad (2.8)$$

for the metric. If we require Poincaré invariance in the world-volume and rotational invariance in the transverse space, we can use the following *Ansatz*

$$\begin{aligned} ds^2 &= B^2(r) \eta_{\alpha\beta} dx^\alpha dx^\beta + F^2(r) \delta_{ij} dx^i dx^j \\ \varphi &= \varphi(r) \end{aligned} \quad (2.9)$$

and then solve for the functions $B(r)$, $F(r)$ and $\varphi(r)$. Using this *Ansatz*, the field equations (2.7) and (2.8) become

$$\begin{aligned} \varphi'' + \left(\xi' + \frac{8-p}{r} \right) \varphi' &= 0 \\ (\log B)'' + \left(\xi' + \frac{8-p}{r} \right) (\log B)' &= 0 \\ (\log F)'' + \left(\xi' + \frac{8-p}{r} \right) (\log F)' + \frac{\xi'}{r} &= 0 \end{aligned} \quad (2.10)$$

$$\begin{aligned} \xi'' + (\log F)'' - \left(\xi' - \frac{8-p}{r} \right) (\log F)' \\ + (p+1) [(\log B)']^2 + (7-p) [(\log F)']^2 + (\varphi')^2 = 0 \end{aligned}$$

where $' \equiv d/dr$, and

$$\xi \equiv (p+1) \log B + (7-p) \log F \quad . \quad (2.11)$$

The general solution of these equations can be easily deduced from the analysis of [26]⁴ and depends on several integration constants which can be uniquely fixed by imposing the asymptotic behavior (2.5) dictated by the boundary state. If we introduce the harmonic functions

$$f_{\pm}(r) = 1 \pm x \frac{\widehat{Q}_p}{r^{7-p}} \quad , \quad (2.12)$$

the non-BPS D-brane solution can be written in a rather simple form and reads

$$\begin{aligned} B^2(r) &= \left(\frac{f_-(r)}{f_+(r)} \right)^{\lambda} \\ F^2(r) &= f_-(r)^{\mu_-} f_+(r)^{\mu_+} \\ e^{\varphi(r)} &= \left(\frac{f_-(r)}{f_+(r)} \right)^{\nu} \end{aligned} \quad (2.13)$$

where

$$\begin{aligned} \lambda = \frac{7-p}{16x} \quad , \quad \mu_{\pm} = \frac{2}{7-p} \pm \frac{p+1}{16x} \quad , \quad \nu = \frac{p-3}{8x} \quad , \\ x = \sqrt{\frac{7-p}{8(8-p)}} \quad . \end{aligned} \quad (2.14)$$

It is not difficult to realize that the metric described by (2.13) possesses a curvature singularity at $r_p \equiv (x \widehat{Q}_p)^{1/(7-p)}$ [29]. Thus, the solution (2.13) is meaningful only in the “physical” region $r > r_p$.

The problem that we want to address now is the consistency of the geometrical description (2.13) with the microscopic string interpretation of the non-BPS D-branes. In other words we want to check whether the classical solution is consistent with the approximations that lead to the action from which it descends. In this respect, we observe that (2.4) is a valid effective action only when curvature effects are small with respect to the string scale so that higher derivative terms can be

⁴See also [29] for a related discussion.

consistently neglected in the Lagrangian. In our case this happens when M_p is large. However, as we have seen at the beginning of this section, the microscopic interpretation implies that the tension M_p is not a free parameter, and the only way to make it large is to consider a superposition of N D-branes and then take the limit $N \rightarrow \infty$. But this is possible only if the D-branes do not interact with each other, *i.e.* if they satisfy a no-force condition.

Unfortunately, the unstable non-BPS D-branes in ten dimensions do not enjoy this property. To see this, let us compute the interaction energy Γ between two D-branes, which, from the closed string point of view, is simply given by

$$\Gamma = \langle Dp | \mathcal{P} | Dp \rangle \quad (2.15)$$

where \mathcal{P} is the closed string propagator. Using standard techniques, it is easy to see that Γ is not vanishing; moreover, by taking the field theory limit, one may find that

$$\Gamma \Big|_{\alpha' \rightarrow 0} = \widehat{M}_p V_{p+1} \frac{\widehat{Q}_p}{r^{7-p}} \quad (2.16)$$

where V_{p+1} is the (infinite) world-volume of the Dp -brane. Eq. (2.16) explicitly shows that there exists a non-vanishing force between two non-BPS D-branes: in fact the attraction due to the exchange of gravitons and dilatons is not compensated by any repulsion because these branes do not carry any charge. Thus, according to our previous discussion, we can conclude that, since it does not satisfy the no-force condition, the classical solution (2.13) is acceptable (for $r > r_p$) as long as we do not require a microscopic string interpretation of the underlying theory.

One may wonder whether these conclusions may change by taking into account the presence of tachyons on the brane world-volume. As is well-known, these fields have non-trivial consequences on the open-string dynamics and modify the structure of the boundary action [18]. Even if these effects are taken into account, and consequently the form of the solution is changed (see for example [29] for a recent discussion on this point), the no-force condition still cannot be satisfied. Furthermore, if we appeal to the existence of tachyonic modes, a more fundamental question arises, namely to what extent a classical geometry can be associated to an unstable system.

In conclusion we see that the two essential requirements for the existence of a consistent geometrical description of a D-brane are its stability and the validity of the no-force condition. As we have seen, these two properties are not satisfied by the non-BPS D-branes considered so far. However, it turns out that in suitable

orbifold compactifications of type II theories, there exist non-BPS D-branes which are stable, *i.e.* tachyon free, and do not interact pairwise, at one-loop, [22]. These are therefore the natural candidates to be considered for a classical supergravity interpretation consistent with a microscopic string description. The study of these branes will be the subject of the remaining part of this paper.

3 Low-energy actions

In this section we provide the necessary ingredients to analyze the geometry associated to the stable non-BPS D-branes in six dimensions. These are non-perturbative configurations of the type II string compactified on T_4/\mathbf{Z}_2 orbifolds, which have been extensively studied using the boundary state formalism [22, 20, 23]. Here we will focus on the simplest case, namely the stable non-BPS D-particle. From its boundary state description it is easy to realize that such a particle is a source for a graviton, a dilaton, four scalars and a vector potential in six dimensions. Our goal is to determine the classical configuration for these fields and study its consistency. Therefore, in subsection 3.1 we first derive the bulk action that governs the dynamics of the fields emitted by the D-particle. As we will see, this is a consistent truncation of the $D = 6$ supergravity action. Later, in subsection 3.2 we will derive the boundary action that describes the couplings of the bulk fields with the D-particle.

3.1 Bulk theory

The theory we consider is the non-chiral supergravity in six dimensions with sixteen supercharges. To describe it, we start from the ten dimensional type IIA string compactified on a torus T_4 and orbifolded with a discrete parity \mathbf{Z}_2 generated by the reflection \mathcal{I}_4 of the four compact directions (labeled by indices $a, b = 6, \dots, 9$). Equivalently, we could start from the type IIB string compactified on the orbifold generated by $\mathcal{I}_4(-1)^{FL}$, where $(-1)^{FL}$ is the operator that changes the sign of all R-R and R-NS states. These two theories are related to each other by a T -duality along one of the compact directions, and thus yield the same low-energy Lagrangian. Both orbifolds have been extensively studied in the literature (see for example [30]) and it is well known that their massless spectrum is described by the six dimensional $(1, 1)$ supergravity coupled to 20 $U(1)$ vector multiplets. The action of this theory

is commonly written in the following compact form

$$S \sim \int d^6x \sqrt{-\det g} e^{-2\varphi} \left[\mathcal{R}(g) + 4 \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{1}{4} (M^{-1})_{IJ} F_{\mu\nu}^I F^{J\mu\nu} + \frac{1}{8} \text{Tr} (\partial_\mu M \partial^\mu M^{-1}) + \dots \right], \quad (3.1)$$

where $g_{\mu\nu}$ is the string frame metric, φ is the dilaton, $H_{\mu\nu\rho}$ is the field strength of the NS-NS two-form, and M is the matrix parameterizing the coset manifold of the scalar fields. In our case, this coset is $SO(4, 20)/SO(4) \times SO(20)$, which has the right dimension to accommodate the 80 scalars of 20 vector multiplets. Finally, $F_{\mu\nu}^I$ contains the field strengths of all the $U(1)$'s present in the spectrum and transforms as a vector under T -duality⁵.

The action (3.1) explicitly displays the full T -duality invariance of the theory. However, for our purposes this form is too general and blurs a few crucial details. First of all, since our aim is to study the theory in the region of its moduli space corresponding to the orbifold T_4/\mathbf{Z}_2 , we are not interested in having a manifestly T -dual invariant formulation. Indeed, the expectation values of the scalars change under T -duality, and so does the shape of the compact space. Secondly, we need to know the precise normalizations of the various terms in the action, and in particular their dependence on the moduli. In fact, from the microscopic description we know that the stability of the non-BPS branes crucially depends on the radii of the compact space [22]. Moreover, if we want to construct from these branes a macroscopic object, the shape of the internal T_4 must be further restricted by fixing all radii to some critical value [8]. For these reasons, we need to write the supergravity action (3.1) in a form capable to make more explicit its relation with the orbifold construction. This means that we must break the $SO(4, 20)/SO(4) \times SO(20)$ invariance and select, out of the 80 scalars, 4 fields that describe the characteristic lengths of the internal space. As will become clear later, we will also need to know the relationship between the fields appearing in the supergravity action and their string counterparts which are associated to the massless vertex operators of the orbifold conformal field theory.

It would be interesting to perform these steps directly within the supergravity context, but this turns out to be a rather non-trivial task. Therefore, we take a different route and exploit the S -duality that relates our model to the heterotic string compactified on a torus T^4 . In this way, we can carry out the reduction

⁵Note that, in general, a T -duality transformation mixes fields which are in different multiplets.

from ten to six dimensions on the heterotic side by using the standard machinery of toroidal compactification, and then translate the result in the type II theory by using the duality map. Notice that this short-cut is possible because this S -duality in six dimensions acts trivially on the moduli spaces of the two theories. This means that all couplings involving the 80 scalars and their dependence on the internal radii do not change in going from the heterotic to the type II string.

We now sketch the derivation of the type II effective action starting from the heterotic string compactified on a 4-torus whose low-energy action (in the string frame) is

$$\begin{aligned}
S_h &= \frac{(2\pi\sqrt{\alpha'})^4}{2\kappa_{10}^2} \int d^6x \sqrt{-\det g^h} e^{-2\varphi^h} \left[\mathcal{R}(g^h) + 4\partial_\mu \varphi^h \partial^\mu \varphi^h + \frac{1}{4} \partial_\mu \phi_a^h \partial^\mu (\phi_a^h)^{-1} \right] \\
&- \frac{(2\pi\sqrt{\alpha'})^4}{4g_{10}^2} \int d^6x \sqrt{-\det g^h} e^{-2\varphi^h} F_{\mu\nu}^I F^{I\mu\nu} + \dots
\end{aligned} \tag{3.2}$$

where the gravitational and the gauge couplings are related in the usual way $\kappa_{10}/g_{10} = \sqrt{\alpha'}/2$. Note that (3.2) is only a subset of the whole action coming from the toroidal compactification since most of the original fields have been put to zero. We will discuss the validity of this truncation after having translated the action (3.2) in the type II language; its motivations will become clearer when the boundary action related to the non-BPS brane is discussed (that is, when the source term is taken into account). Here, we just notice that in (3.2) only the scalars related to dilatations of the compact dimensions have been explicitly written: since we consider a compactification where the torus is just a product of four orthogonal circles, these scalars are simply the four diagonal components of the ten-dimensional metric in the internal space, *i.e.* $g_{aa}^h \equiv \phi_a^h$ with $a = 6, \dots, 9$, and the dilaton φ^h . The v.e.v.'s of these scalar fields are given by

$$\langle \phi_a^h \rangle = \frac{(R_a^h)^2}{\alpha'} \quad \text{and} \quad e^{\langle \varphi^h \rangle} = \frac{\alpha'}{V_h^{1/2}} g' , \tag{3.3}$$

where g' is the heterotic string coupling constant, and $V_h \equiv \prod_{a=6}^9 R_a^h$. In writing (3.2) we have also assumed that the gauge group is broken to $U(1)^{16}$ by suitable Wilson lines, and $F_{\mu\nu}^I$ denotes the surviving field strengths ($I = 1, \dots, 16$).

The correspondence between the heterotic and the type II theories can be established by means of the following chain of dualities

$$\text{Het. } T^4 \xLeftrightarrow{S} \text{Type I } T^4 \xLeftrightarrow{4T} \text{IIB } \frac{T^4}{\Omega \mathcal{I}_4} \xLeftrightarrow{S} \text{IIB } \frac{T^4}{(-1)^{F_L} \mathcal{I}_4} \left(\xLeftrightarrow{T} \text{IIA } \frac{T^4}{\mathcal{I}_4} \right) . \tag{3.4}$$

The last step is not really necessary for our purposes, but it is useful to have it in mind. In fact, for some practical calculations the type IIB picture is easier, whereas the geometrical interpretation is clearer for the orbifold of type IIA which is a singular limit of a smooth K3 manifold. In Figure 2 we briefly summarize how the bosonic fields of this theory emerge from three different points of view.

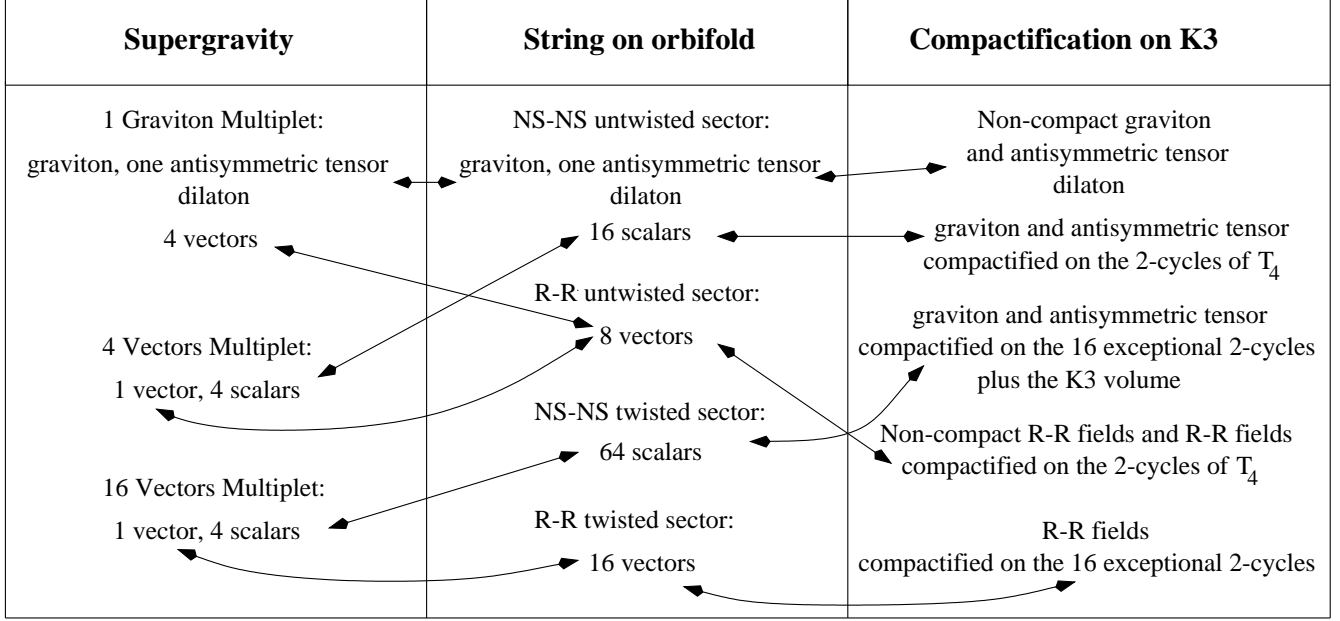


Figure 2: The bosonic low energy spectrum from three different points of view

By following the various steps of (3.4), one can see how the parameters of the different theories are related to each other [30]. For instance, the radii R_a^B and the string coupling constant g in the type IIB orbifold are related to the corresponding quantities of the original heterotic theory as

$$R_a^B = \frac{\sqrt{2} V_h^{1/2}}{R_a^h} \quad , \quad g = \frac{\sqrt{2} V_h}{\alpha'^2} \frac{1}{g'} \quad . \quad (3.5)$$

By performing a further T -duality in one of the four compact directions (say x^9) we can reach the type IIA orbifold, for which we have

$$R_a^A = \frac{\sqrt{2} V_h^{1/2}}{R_a^h} \quad \text{for } a \neq 9 \quad , \quad R_9^A = \frac{R_9^h \alpha'}{\sqrt{2} V_h^{1/2}} \quad , \quad g = \frac{R_9^h V_h^{1/2}}{\alpha'^{3/2}} \frac{1}{g'} \quad . \quad (3.6)$$

The numerical coefficient in these relations have been fixed by checking that the masses of BPS objects take the expected values after a duality transformation. This

is the same derivation used in [14]; however, here we do not follow their conventions and our results are slightly different. We keep the string length fixed, *i.e.* $\alpha'_h = \alpha'_B = \alpha'_A \equiv \alpha'$, and define the dilaton v.e.v. in the orbifold compactification as

$$e^{\langle \varphi^B \rangle} = \frac{\sqrt{2} \alpha'}{V_B^{1/2}} g , \quad (3.7)$$

$V_B = \prod_{a=6}^9 R_a^B$, and similarly for the IIA case. The factor of $\sqrt{2}$ in the above definition is quite natural. In fact, as usual, the dilaton v.e.v. in a compactified theory contains the volume of the compact space. In the toroidal case one has $\text{Vol} \sim V$, while in the orbifold the \mathbf{Z}_2 identification halves the “physical” volume of the internal space: $\text{Vol} \sim V/2$.

Recalling that the radii are related to the v.e.v. of the four scalar fields ϕ_a , and using (3.3) and (3.7), we can lift the above duality maps to the field level and obtain

$$\varphi^{A,B} = -\varphi^h , \quad \phi_a^{A,B} = 2 \frac{\sqrt{\prod_{b=6}^9 \phi_b^h}}{\phi_a^h} . \quad (3.8)$$

Finally, by exploiting the invariance of the metric in the Einstein frame under S -duality, one can find the usual relation between the string-frame metrics:

$$g_{\mu\nu}^{A,B} = e^{-2\varphi^h} g_{\mu\nu}^h . \quad (3.9)$$

Equipped with this machinery, we are ready to perform the S -duality on the heterotic action (3.2), and rewrite it in terms of IIB quantities. Using Eq.s (3.3), (3.8) and (3.9), we get

$$\begin{aligned} S_B &= \frac{(2\pi\sqrt{\alpha'})^4}{2\kappa_{10}^2} \int d^6x \sqrt{-\det g^B} e^{-2\varphi^B} \left[\mathcal{R}(g_B) + 4\partial_\mu \varphi^B \partial^\mu \varphi^B + \frac{1}{4} \partial_\mu \phi_a^B \partial^\mu (\phi_a^B)^{-1} \right] \\ &- \frac{(2\pi\sqrt{\alpha'})^4}{4g_{10}^2} \int d^6x \sqrt{-\det g^B} F_{\mu\nu}^I F^{I\mu\nu} + \dots . \end{aligned} \quad (3.10)$$

It is not difficult to see that this action is consistent with the perturbative string amplitudes that can be calculated in the IIB orbifold. However, in order to do this comparison one has first to rewrite the Lagrangian (3.10) in the Einstein frame by rescaling the metric $g_{\mu\nu} = e^{\tilde{\varphi}} G_{\mu\nu}$. Here $\tilde{\varphi} = (\varphi - \varphi_\infty)$, where φ_∞ is the constant value of the dilaton at spatial infinity, which in our case is simply the v.e.v. defined in (3.7). Another rescaling is usually done on the gauge fields. In fact, in type II theory these are taken to be dimensionless regardless of the number of indices they carry, while on the heterotic side they have canonical dimensions. Thus, we

introduce $\tilde{F} = 2\sqrt{\alpha'} g F$. Finally, for later convenience, we write $\phi_a^B = \langle \phi_a^B \rangle e^{2\tilde{\eta}_a^B}$. In terms of these rescaled fields, the action (3.10) becomes

$$S_B = \frac{1}{2\kappa_{orb}^2} \int d^6x \sqrt{-\det G_B} \left[\mathcal{R}(G_B) - \partial_\mu \tilde{\varphi}^B \partial^\mu \tilde{\varphi}^B - \partial_\mu \tilde{\eta}_a^B \partial^\mu \tilde{\eta}_a^B \right. \\ \left. - \frac{1}{4} \left(\frac{2\kappa_{orb}^2}{g_{orb}^2} \right) e^{\tilde{\varphi}^B} \tilde{F}_{\mu\nu}^I \tilde{F}^{I\mu\nu} \right] , \quad (3.11)$$

where

$$\kappa_{orb}^2 = \frac{2\kappa_{10}^2 g^2}{(2\pi)^4 V_B} , \quad \text{and} \quad \frac{2\kappa_{orb}^2}{g_{orb}^2} = \frac{\alpha'^2}{4 V_B} . \quad (3.12)$$

Contrary to what happens in the heterotic theory, here the gravitational and gauge couplings have a different dependence on the radii of the compact space. This fact can be naturally understood by comparing the tree-level string amplitudes with the vertices derived from the field theory Lagrangian. As usual, the moduli dependence of the couplings is directly related to the “nature” of strings involved in the amplitudes. Since in the heterotic theory both the gauge and the gravitational fields are made out of the same kind of closed strings, it is natural that all couplings have a uniform dependence on the moduli. On the contrary, in the type II setup, the gauge fields we are looking at come from the twisted R-R sector. In this case the mode expansion of the string coordinates does contain momentum along the compact directions. Because of this, the twisted and untwisted vacua are differently normalized

$$_U \langle p_1, n | p_2, m \rangle_U \sim \delta^{(6)}(p_1 - p_2) \delta_{nm} V , \quad _T \langle p_1, 0 | p_2, 0 \rangle_T \sim \delta^{(6)}(p_1 - p_2) ,$$

and this difference reflects on the volume dependence of the gauge and gravitational couplings (3.12). It is interesting to remark that the gauge kinetic term in (3.11) becomes canonically normalized only for a particular value of the radii of the internal space, namely at $R_a = \sqrt{\alpha'}/2$. As we will see in the next paragraph, this value plays a privileged role also from a different point of view.

We finally comment on the consistency of the truncation we did in deriving the effective action (3.11). As we already mentioned, we have considered only those massless fields which couple directly to the non-BPS D-particle we want to study, and switched off all other fields. One may ask whether this truncation is consistent with the equations of motion of the complete theory [31]. In particular, problems can arise if in the full Lagrangian there are interaction terms which are only linear in one of the fields here disregarded, for instance the twisted NS-NS scalars, call them ξ . In this case, the equations of motion for ξ will contain a term which is not

automatically vanishing in our approximation, since it is independent of the field itself and a contradiction may arise. However, it can be checked with perturbative arguments that these terms cannot be present in the complete Lagrangian. For instance, the twisted NS-NS scalars ξ are described by vertex operators containing a left and a right spin field of the internal space. Thus, they have a non-zero M -point amplitude, only if one of the other external vertices also contains these spin fields. However, this is not the case for the vertices corresponding to the fields we were considering. This shows that the above mentioned problems cannot arise with the twisted NS-NS scalars and that we can safely set them to zero. Similar world-sheet considerations can be done also for the R-R fields we switched off. For the untwisted scalars, like the compact off-diagonal entries of ten dimensional metric g_{ab} , it is easier to look directly at the part of the action where they appear. In fact, as well known, the terms containing only untwisted fields can be derived by means of the toroidal compactification from the original ten dimensional description: in the usual calculation, one has simply to put to zero all the fields which are odd under the orbifold operation. In this way one can check that the untwisted scalars appear at least quadratically in the action, and thus our truncation is consistent.

3.2 Boundary action

From the supergravity point of view, the boundary action is seen as the source term which one must add to the bulk action in order to describe a D-brane configuration. In string theory, this source can be efficiently represented by a boundary state and, in particular, by its overlaps with the massless closed string states. The boundary states $|Dp\rangle$ that describe the non-BPS Dp -branes of the type II orbifolds, have been studied in detail in the literature [22, 20]. Here we just recall the main features that will be employed in the following section.

A first important point is that $|Dp\rangle$ is non-trivial only in the NS-NS untwisted and R-R twisted sectors of the theory [13, 8, 12]. In particular, focusing on the non-BPS D-particle present in the type IIB/ \mathbf{Z}_2 orbifold, one has

$$|D0\rangle = |B0\rangle_{\text{NS-NS},U} + |B0\rangle_{\text{R-R},T_I} \quad , \quad (3.13)$$

where the index $I = 1, \dots, 16$ in the twisted part indicates on which orbifold plane the D-brane is placed. The explicit form of the coherent states $|B0\rangle$ in (3.13) and their overlaps with perturbative closed string states have been studied in [23]. This paper, however, uses fields and vertex operators that are essentially written in the

framework of the original string theory in ten dimensions. But, if one wants to make contact with the six dimensional bulk theory discussed in the previous subsection, one must use more appropriate fields. These can be easily obtained by observing, for example, that the vertex operators that describe the six-dimensional graviton and dilaton have the same structure of their ten-dimensional analogues, but contain only oscillators with indices in the non-compact directions. The internal part of the ten-dimensional vertices describes instead the six dimensional scalars. Keeping this in mind, it is not difficult to find the relation between the canonically normalized fields $h'_{\mu\nu}$ and φ' used in [23] and the canonically normalized fields $\hat{h}_{\mu\nu}$, $\hat{\eta}_a$, $\hat{\varphi}$ to be used in six dimensions. This relation is

$$\begin{aligned}\hat{h}_{\mu\nu} &= h'_{\mu\nu} - \eta_{\mu\nu} \left[\frac{\sqrt{2}}{2} \varphi' - \frac{1}{4} \sum_a h'_{aa} \right] , \\ \hat{\eta}_a &= h'_{aa} - \frac{1}{2\sqrt{2}} \varphi' , \\ \hat{\varphi} &= \frac{3\sqrt{2}}{2} \varphi' - \frac{1}{2} \sum_a h'_{aa} .\end{aligned}\tag{3.14}$$

Writing the vertex operators associated to the hatted field in (3.14) and to the (canonically normalized) twisted R-R potential \hat{A}_μ^I , we can compute the overlaps with the boundary state (3.13) and find the couplings between the bulk fields and the D-particle. Using the results of [23] and the redefinitions (3.14), we get

$$\begin{aligned}\langle D0|\hat{h}_{\mu\nu}\rangle &= M_0 V_1 \kappa_{orb} \hat{h}_{00} , \quad \langle D0|\hat{\varphi}\rangle = \frac{M_0 V_1}{2} \kappa_{orb} \hat{\varphi} , \\ \langle D0|\hat{\eta}_a\rangle &= \frac{M_0 V_1}{2} \kappa_{orb} \hat{\eta}_a , \quad \langle D0|\hat{A}_\mu^I\rangle = \sqrt{\frac{8 V_B}{\alpha'^2}} M_0 V_1 \kappa_{orb} \hat{A}_0^I ,\end{aligned}\tag{3.15}$$

where

$$M_0 = \frac{\sqrt{2} T_0}{(2\pi)^2 \kappa_{orb} V_B^{1/2}} = \frac{1}{\sqrt{\alpha'} g}\tag{3.16}$$

is the mass of the non-BPS D-particle and V_1 is the (infinite) length of its world-line. The overlaps (3.15) represent the one-point functions of the bulk fields encoded in the boundary action. To write it we find convenient to use the same notation of the previous subsection, and not to work any more with the canonically normalized hatted fields. The relation between the latter and the fields appearing in the bulk Lagrangian (3.11) is simply given by

$$G_{\mu\nu}^B = \eta_{\mu\nu} + 2 \kappa_{orb} \hat{h}_{\mu\nu} , \quad \hat{\varphi} = \frac{\tilde{\varphi}^B}{\kappa_{orb}} , \quad \hat{\eta}_a = \frac{\eta_a^B}{\kappa_{orb}} , \quad \hat{A}_\mu^I = \frac{\tilde{A}_\mu^I}{g_{orb}} ,\tag{3.17}$$

where κ_{orb} and g_{orb} are defined in (3.12). Then, it is easy to realize that the overlaps (3.15) are consistent with the following boundary action

$$S_{\text{boundary}} = -M_0 \int d\tau e^{-\frac{1}{2}\tilde{\varphi}^B - \frac{1}{2}\sum_a \tilde{\eta}_a^B} \sqrt{-G_{00}^B} + M_0 \int d\tau \tilde{A}_0^I. \quad (3.18)$$

Of course, this action does not describe the complete world-volume dynamics of the non-BPS D-particle. In fact, in deriving it we considered only the trivial configuration for the fields related to *open* strings and we switched off all non-linear couplings with *closed* strings⁶. However, for our purposes the action (3.18) will be sufficient. Note that its untwisted part can be obtained also from the action (2.3) of the unstable non-BPS branes discussed in Section 2: one has just to perform a toroidal compactification and remove all fields that are odd under the orbifold projections. On the other hand, the twisted part of (3.18) accounts for the minimal coupling of a charged particle with its gauge field. Notice that the strength we found for this coupling is consistent with the *S*-dual interpretation of the D-particle. In fact, on the heterotic side this particle corresponds to a perturbative massive state with charge $q_h = 2$ under one of the 16 unbroken $U(1)$'s. Of course, this $U(1)$ charge does not change in the duality map and, thus, translating the heterotic result in type II language, one should find agreement with the overlap (3.15). This is indeed the case; in fact, taking into account the relation between the type II gauge fields \tilde{A} and those of the heterotic theory introduced after Eq. (3.10), we can see that the gauge charge $q_h = 2$ becomes exactly the one that we read from (3.18).

So far we have discussed the boundary action for a single non-BPS brane. However, in order to form a macroscopic object one should consider a superposition of many branes. Only in this case, in fact, one can hope that the source creates a smooth classical geometry where it is possible to neglect string and loop corrections. Of course, the dynamics of many coincident branes is quite complicated and can radically change the couplings previously derived for a single object. For BPS configurations, this is not the case because the D-branes do not interact with each other. Thus the effect of the superposition of N D-branes is simply to multiply by N the strength of all couplings. In a non-supersymmetric setup, instead, the branes in general interact among them in a non-trivial way. However, as shown in [22], the non-BPS D-particles of the IIB/ \mathbf{Z}_2 orbifold enjoy two fundamental properties that make them similar to the usual BPS D-branes. In fact, the orbifold projection always kills the tachyon zero-mode and, if $R_a^B \geq \sqrt{\alpha'}/2$ the winding excitations of

⁶For instance, anomalous couplings, similar to those of usual D-branes, are present also for non-BPS branes [19]

the tachyon, which survive the projection, have a positive mass². Thus, the non-BPS D-particles are stable. Moreover, if $R_a^B = \sqrt{\alpha'/2}$ (which is also the value where the gauge kinetic term in (3.11) becomes canonically normalized), the “would-be” tachyons become massless and an accidental Bose-Fermi degeneracy appears in the spectrum. Thus, at the critical radii the force between two non-BPS D-particles vanishes at one-loop [22], *i.e.*

$$\Gamma = \langle D0 | \mathcal{P} | D0 \rangle = 0 . \quad (3.19)$$

For this reason, it is natural to conjecture that, in this particular case, it is possible to describe N non-BPS D-particles simply by taking the naïve sum of N boundary states previously introduced to describe a single object, that is

$$|D0, N\rangle = N |D0\rangle . \quad (3.20)$$

We want to stress that the no-force condition (3.19) is clearly a necessary ingredient for this simplification to hold, but it is not sufficient to really prove the validity of the assumption (3.20). In fact, Eq. (3.19) proves the vanishing of the interaction only at one-loop (from the open-string point of view), and does not guarantee that a similar result occurs at higher loops. In other words, from Eq. (3.19) we can see that the interactions between N D-particles vanish at the leading order in N , while we know that the supergravity description is reliable in the opposite regime, $N \rightarrow \infty$. Thus, we take Eq. (3.20) as a working hypothesis and, in the next section, we will check whether this leads to acceptable space-time configurations for the metric and the other fields.

4 The non-BPS D-particle solution

In the previous section we have shown that the action describing the dynamics of the fields emitted by a non-BPS D-particle in six dimensions, is given by the sum of Eq.s (3.11) and (3.18) which we rewrite here in a simplified notation

$$\begin{aligned} S = & \frac{1}{2\kappa_{orb}^2} \int d^6x \sqrt{-\det G} \left[\mathcal{R}(G) - \partial_\mu \varphi \partial^\mu \varphi - \partial_\mu \eta_a \partial^\mu \eta_a - \frac{1}{4} e^\varphi F_{\mu\nu} F^{\mu\nu} \right] \\ & - M \int d\tau e^{-\frac{1}{2}\varphi - \frac{1}{2}\sum_a \eta_a} \sqrt{-G_{00}} + M \int d\tau A_0 . \end{aligned} \quad (4.1)$$

Notice that here we have fixed the compact volume to its critical value $V_c = \alpha'^2/4$ where the no-force condition holds at one loop, and, according to our working

hypothesis (3.20), we have put $M = N M_0$. The field equations that follow from this action are ⁷

$$\frac{1}{\sqrt{-\det G}} \partial_\mu \left(\sqrt{-\det G} G^{\mu\nu} \partial_\nu \varphi \right) - \frac{1}{8} e^\varphi F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} T(x) \delta^5(\vec{x}) \quad (4.2)$$

for the dilaton,

$$\frac{1}{\sqrt{-\det G}} \partial_\mu \left(\sqrt{-\det G} G^{\mu\nu} \partial_\nu \eta_a \right) = \frac{1}{2} T(x) \delta^5(\vec{x}) \quad (4.3)$$

for the 4 scalar fields,

$$\partial_\mu \left(\sqrt{-\det G} e^\varphi F^{\mu\nu} \right) = -2 M \kappa_{orb}^2 G^\nu{}_0 \delta^5(\vec{x}) \quad (4.4)$$

for the gauge field, and

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} \mathcal{R} - \left(\partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} G_{\mu\nu} \partial_\rho \varphi \partial^\rho \varphi \right) - \left(\partial_\mu \eta_a \partial_\nu \eta_a - \frac{1}{2} G_{\mu\nu} \partial_\rho \eta_a \partial^\rho \eta_a \right) \\ - \frac{1}{2} e^\varphi \left(F_{\mu\rho} F^\rho{}_\nu - \frac{1}{4} G_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right) = G_{00} G_\mu{}^0 G_\nu{}^0 T(x) \delta^5(\vec{x}) \end{aligned} \quad (4.5)$$

for the metric, where

$$T(x) = -M \kappa_{orb}^2 e^{-\frac{1}{2}\varphi - \frac{1}{2}\sum_a \eta_a} \frac{\sqrt{-G_{00}}}{\sqrt{-\det G}} \quad (4.6)$$

Our task is to find a solution to these equations describing a static and spherically symmetric non-BPS D-particle in which the fields depend only on the distance in transverse space, r . There are several ways to reach this goal. A first possibility is to build up the solution iteratively via a perturbative approach by expressing the various fields as series in powers of $1/r^3$ with arbitrary coefficients (recall that the usual $1/r^{D-p-3}$ dependence of a p -brane in D dimensions reduces in the present case to $1/r^3$). Inserting this *Ansatz* in the coupled system (4.2)–(4.5), one can determine all coefficients by solving the equations order by order in $1/r^3$. For example, up to third order in $1/r^3$ the solution looks like

$$\varphi \simeq \frac{1}{4} \frac{Q}{r^3} - \frac{1}{8} \left(\frac{Q}{r^3} \right)^2 + \frac{1}{24} \left(\frac{Q}{r^3} \right)^3 + \dots \quad (4.7)$$

$$\eta_a \simeq \frac{1}{4} \frac{Q}{r^3} + \dots \quad (4.8)$$

$$A_0 \simeq -\frac{Q}{r^3} + \frac{1}{2} \left(\frac{Q}{r^3} \right)^2 - \frac{1}{3} \left(\frac{Q}{r^3} \right)^3 + \dots \quad (4.9)$$

$$G_{00} \simeq -1 + \frac{3}{4} \frac{Q}{r^3} - \frac{21}{32} \left(\frac{Q}{r^3} \right)^2 + \frac{61}{128} \left(\frac{Q}{r^3} \right)^3 + \dots \quad (4.10)$$

$$G_{ij} \simeq \delta_{ij} \left[1 + \frac{1}{4} \frac{Q}{r^3} - \frac{3}{32} \left(\frac{Q}{r^3} \right)^2 + \frac{5}{384} \left(\frac{Q}{r^3} \right)^3 + \dots \right], \quad (4.11)$$

⁷We use the static gauge $X^0 = \tau$.

where

$$Q = \frac{2 M \kappa_{orb}^2}{3 \Omega_4} \sim N g \alpha'^{3/2} \quad (4.12)$$

and the indices $i, j = 1, \dots, 5$ label the transverse directions.

This same result can be obtained via an alternative approach based on the use of the boundary state, which, as shown in [27, 28], allows to find the asymptotic behavior of the various fields at large distance from the source. For the non-BPS D-particle, this method has been recently used in [23] where the leading terms proportional to Q/r^3 have been obtained, and generalized for any p in [32] (provided the redefinitions (3.14) are taken into account). Actually, one can do more. Remembering that Q is proportional to the 't Hooft coupling $\lambda \sim Ng$ (see Eq. (4.12)), the above expansions for large r can also be interpreted as expansions in λ . Since in string theory different powers of λ characterize open string diagrams of different topologies, the various terms in (4.7)-(4.11) can be associated to the one-point functions of massless bulk fields evaluated on world-sheets with an increasing number of boundaries. Specifically, the terms linear in Q arise from one-point function on a disk diagram, the terms proportional to Q^2 from one-point functions on an annulus, and so on. However, for the purpose of finding the classical solution, it is not really necessary to perform such calculations in string theory, but it is sufficient to do them directly in the low-energy field theory described by the action (4.1). Here one has simply to compute (in configuration space) diagrams like the ones represented in Figure 3, where the couplings with the sources are determined by the boundary part of the effective action and the interaction vertices from its bulk part. Despite

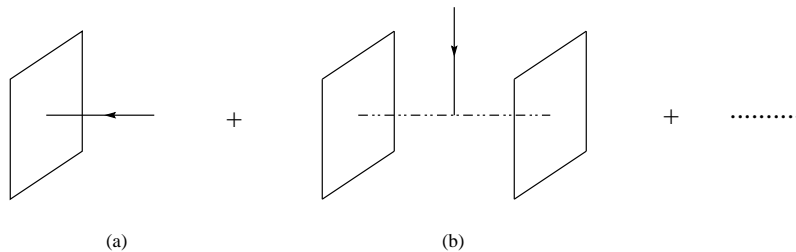


Figure 3: The leading contribution to the one-point function of a bulk field expressed in a diagrammatic way. Diagram (a) yields the leading term in the large distance expansion and is proportional to Q , whereas diagram (b) corresponds to the next-to-leading correction proportional to Q^2 .

its conceptual simplicity, this diagrammatic method requires calculations which become more and more cumbersome as one proceeds in the perturbative expansion.

Nevertheless, it is useful because it clarifies the origin and the meaning of the various terms. In Appendix A we present the detailed calculations of the diagrams that contribute to the classical solution up to Q^2 .

Let us now return to the perturbative expansions (4.7)-(4.11) of the D-particle solution. Differently from what expected [23], it turns out that it is possible to re-sum these series and present the fields in a closed form. Indeed we find

$$\varphi = \frac{1}{4} \ln \left[1 + \sin \left(\frac{Q}{r^3} \right) \right] \quad (4.13)$$

$$\eta_a = \frac{1}{4} \frac{Q}{r^3} \quad (4.14)$$

$$A_0 = -1 + \frac{\cos \left(\frac{Q}{r^3} \right)}{1 + \sin \left(\frac{Q}{r^3} \right)} \quad (4.15)$$

$$G_{00} = - \left[1 + \sin \left(\frac{Q}{r^3} \right) \right]^{-3/4} \quad (4.16)$$

$$G_{ij} = \delta_{ij} \left[1 + \sin \left(\frac{Q}{r^3} \right) \right]^{1/4} . \quad (4.17)$$

It is not difficult, but rather tedious, to check that (4.13)-(4.17) is indeed a solution of the differential equations (4.2)-(4.5). In Appendix B we will provide all details for deriving the above expressions directly from the field equations (4.2)-(4.5), and present also their extension to the case of a generic p -brane.

We now discuss the properties of the solution (4.13)-(4.17). First of all, we observe that it is well-defined only for $r \geq Q^{1/3}$. In fact, in the region $r < Q^{1/3}$ the dilaton, the gauge field and G_{00} have branch cut singularities at

$$r_n = \left[\frac{Q}{(3/2 + 2n)\pi} \right]^{1/3} \quad \text{for } n = 0, 1, 2, \dots \quad (4.18)$$

Moreover, the scalar curvature \mathcal{R} diverges at these singular points. Clearly, when the curvature is big, the classical supergravity description is not any more reliable; moreover, since the singularities at $r = r_n$ are naked, the entire solution is unacceptable according to the cosmic censorship conjecture. This result seems to indicate that the prediction made in [22], about the possibility of having a classical description for stable non-BPS D-branes, does not hold, at least in the case that we have considered here. Let us be more specific about this fact. From the supergravity point of view, Q is a free parameter, and solutions with different values of Q are all on equal footing. Hence they are all unacceptable for the reason we have explained above. However, if we appeal to the underlying string theory, some

crucial differences come into play. First of all, Q is not any more a free parameter since it is related to the fundamental quantities of the microscopic theory as shown in (4.12). Moreover, in a string context one may expect *a priori* that the classical supergravity description can break down at distances of the order of the string scale, where the massless closed string states cease to be good probes for the geometry. Thus, at $r \sim \sqrt{\alpha'}$ stringy effects must be taken into account, and the entire supergravity approximation must be reconsidered. In other words, if the naked singularities of a classical solution are at distances smaller or equal to the string scale, no contradictions arise, and the supergravity solution can be accepted at larger distances.

This is what happens for a single D-particle, *i.e.* $N = 1$. In fact, since the first singularity is at $r_0 = (2Q/3\pi)^{1/3} < Q^{1/3}$, and, for $N = 1$, $Q^{1/3} \sim g^{1/3}\sqrt{\alpha'}$, the region where the classical solution starts to have problems is inside the region in which stringy corrections are relevant. Thus, Eq.s (4.13)-(4.17) represent a valid classical solution associated to a stable non-BPS D-particle in six dimensions at distances much larger than the string scale. However, if we also want to justify the classical approximation and give a reason for disregarding loop corrections in the bulk, we must also take the limit $g \rightarrow 0$. In this case the fields produced by the D-brane become just small fluctuations around the trivial background and the only relevant terms are the leading ones in the Q/r^3 expansion.

Things are different for $N \neq 1$. In fact, as is clear from (4.12), if we increase the value of N , the parameter Q becomes macroscopic (*i.e.* $Q^{1/3} \gg \sqrt{\alpha'}$), and the solution (4.13)-(4.17) exhibits naked singularities also in a region which is not affected by stringy fuzziness effects and where the classical approximation is reliable. Therefore, according to the cosmic censorship conjecture, the solution (4.13)-(4.17) must be rejected, and its source, namely a stack of many *non-interacting* D-particles, must be regarded as non physical.

We would like to stress that our conclusions are not in contradiction with the result of [22] about the vanishing of the force between two non-BPS D-particles at critical radius. In fact, the result of [22] is exact in α' but perturbative in the 't Hooft coupling $\lambda \sim Ng$, and is due to a cancellation occurring at one loop because of an accidental Bose-Fermi degeneracy of the open string spectrum. The classical solution (4.13)-(4.17) is instead valid in a very different regime, since it is perturbative in α' but exact in λ . Therefore, our result should be compared with that of [22] only in the limit $\lambda \rightarrow 0$, and if we do this, we too find a vanishing force at the first order in λ . This can be easily seen by inserting the classical solution

(4.13)-(4.17) into the boundary part of action (4.1). Expanding at first order in Q , and subtracting the vacuum energy, we find

$$\begin{aligned} S_{\text{boundary}} &= -M \int d\tau e^{-\frac{1}{2}\varphi - \frac{1}{2}\sum_a \eta_a} \sqrt{-G_{00}} + M \int d\tau A_0 \\ &\sim -M \int d\tau \frac{Q}{r^3} \left(-\frac{1}{8} - \frac{1}{2} - \frac{3}{8} + 1 \right) = 0 \quad . \end{aligned} \quad (4.19)$$

A similar calculation shows however, that the no-force condition is *not* satisfied at the next-to-leading order. It would be interesting to derive this result also from an open string computation at two loops (see [33] for a recent discussion on this issue). In this case the non-trivial dynamics of the open strings living on the non-BPS D-brane becomes relevant, and due to the lack of supersymmetry one does not expect any cancellation to occur.

We conclude this section with a final observation. The fact that the stable non-BPS D-particles do not satisfy a no-force condition at all orders in λ , is also suggested by the behavior of G_{00} around $r \sim Q^{1/3}$. One can check that, before reaching the first singularity at r_0 , the derivative of G_{00} changes sign in $r = r_G > r_0$, see Figure 4. As is well known, this fact indicates that something strange

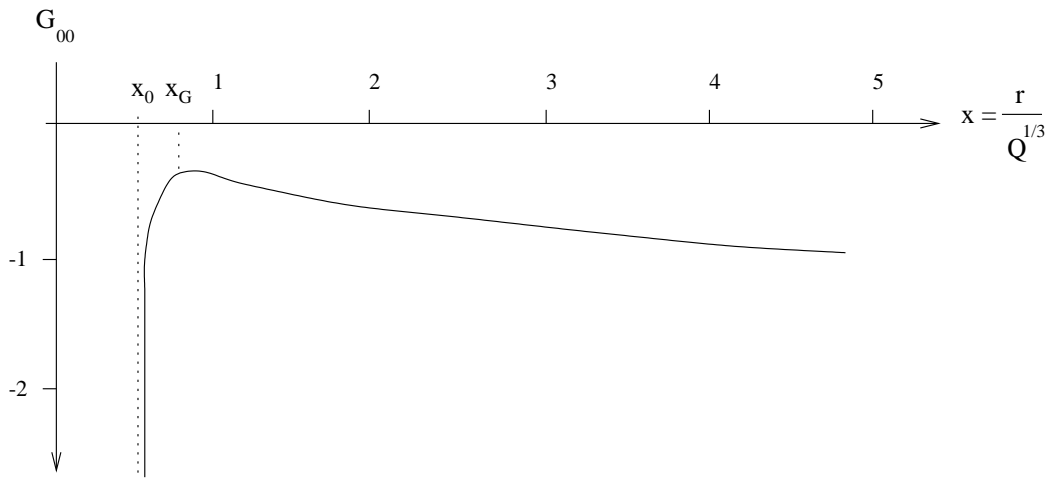


Figure 4: This is the behavior of G_{00} in the region around $x \equiv r/Q^{1/3} = 1$. The derivative of G_{00} changes sign around $x_G \sim 0.8$, while the first singularity (the repulson) is at $x_0 \sim 0.6$.

is happening: indeed, the gravitational force changes its sign at $r = r_G$, and the singularity located at $r = r_0$ (where $G_{00} \rightarrow -\infty$) can be called *repulson* [24], because close to it, massive particles repel each other. These naked singularities have been

recently studied in the context of string theory in [25] where a mechanism for their resolution has been proposed. Since our configuration exhibits properties similar to those discussed in [25], it would be interesting to see whether the same kind of mechanism works also in our case and resolves the singularity we have found.

5 General solution and discussion

As we have stressed several times, the explicit form of the solution (4.13)-(4.17) crucially depends on the detailed knowledge of the source term, and in particular on the hypothesis (3.20). In this last section, we relax this assumption and consider a more general solution of the field equations (4.2)-(4.5). In fact, on general grounds, one may expect that there exist more complicated configurations of non-BPS D-particles which are stable and do not display any pathological behavior in the corresponding classical geometry. For example, one can think of a non-trivial bound state of non-BPS D-particles described by some complicated boundary state $|B\rangle$ and, correspondingly, by a boundary action which could differ from (3.18) even in its functional dependence on the fields.

To explore these possibilities, we therefore study the general solution of the differential equations (4.2)-(4.5) under the minimal amount of requirements. We simply ask that the solution describe an asymptotically flat geometry, be spherically symmetric in the transverse directions and also that all scalar fields have vanishing v.e.v. at infinity. Instead, we do not enforce any specific behavior on the leading terms in the large distance expansion. In fact, as we have explicitly seen in the previous section, these are directly related to the specific microscopic structure of the source. Our only hypotheses about it are therefore that it couples to the graviton, the dilaton, the scalars η_a and to one twisted R-R gauge potential. As we have shown in Section 3, this set of fields defines a consistent truncation of the full six-dimensional supergravity theory.

Under these assumptions, we now study the most general solution of the field equations (4.2)-(4.5), after removing the source terms in the right hand sides. The resulting homogeneous equations can be analyzed by generalizing the methods of [26] and the general solution can be written in a closed form in terms of elementary functions. It will depend on some integration constants (two for each equation); half of them are fixed by the general requirements discussed above, and the remain-

ing ones are free parameters which can be associated to the microscopic structure of the source. In Appendix B we will provide the details to solve explicitly the homogeneous field equations (4.2)-(4.5); here we simply write the result. To do this, it is first convenient to introduce the functions

$$\begin{aligned} f_{\pm}(r) &= 1 \pm x \frac{Q}{r^3} \\ X(r) &= \alpha + \beta \ln \left(\frac{f_-(r)}{f_+(r)} \right) \end{aligned} \quad (5.1)$$

where Q is defined in (4.12), and α, β and x are constants. Then, the general solution gets the following form

$$e^{\eta_a} = \left(\frac{f_-(r)}{f_+(r)} \right)^{\delta} \quad (5.2)$$

$$e^{2\varphi} = \left(\frac{\cosh X(r) + \gamma \sinh X(r)}{\cosh \alpha + \gamma \sinh \alpha} \right) \left(\frac{f_-(r)}{f_+(r)} \right)^{\frac{3}{4}\epsilon} \quad (5.3)$$

$$A_0 = \sqrt{2(\gamma^2 - 1)} \left(\frac{\sinh X(r) (\cosh \alpha + \gamma \sinh \alpha)}{\cosh X(r) + \gamma \sinh X(r)} - \sinh \alpha \right) \quad (5.4)$$

$$G_{00} = - \left(\frac{\cosh X(r) + \gamma \sinh X(r)}{\cosh \alpha + \gamma \sinh \alpha} \right)^{-\frac{3}{2}} \left(\frac{f_-(r)}{f_+(r)} \right)^{\frac{3}{8}\epsilon} \quad (5.5)$$

$$G_{ij} = \delta_{ij} \left(\frac{\cosh X(r) + \gamma \sinh X(r)}{\cosh \alpha + \gamma \sinh \alpha} \right)^{\frac{1}{2}} (f_-(r))^{\frac{2}{3} - \frac{1}{8}\epsilon} (f_+(r))^{\frac{2}{3} + \frac{1}{8}\epsilon} \quad (5.6)$$

where

$$\epsilon = \pm \frac{4}{3} \sqrt{4 - 3\beta^2 - 12\delta^2} \quad . \quad (5.7)$$

As anticipated, this solution depends on some integration constants, namely $\alpha, \beta, \gamma, \delta$, and x , which can be fixed by specifying the form of the source term. For example, if we impose the boundary conditions corresponding to N coincident and non-interacting D-particles, which was the physical situation considered in Section 4, namely if we require that at large distance the fields behave as

$$\begin{aligned} \varphi &\simeq \frac{1}{4} \frac{Q}{r^3} + \dots \quad , \quad \eta_a \simeq \frac{1}{4} \frac{Q}{r^3} + \dots \quad , \quad A_0 \simeq -\frac{Q}{r^3} + \dots \quad , \\ G_{00} &\simeq -1 + \frac{3}{4} \frac{Q}{r^3} + \dots \quad , \quad G_{ij} \simeq \delta_{ij} \left(1 + \frac{1}{4} \frac{Q}{r^3} + \dots \right) \quad , \end{aligned} \quad (5.8)$$

one finds that the constants β, γ, δ , and x must be chosen as

$$x\beta = \pm \frac{i}{4} \quad , \quad \gamma = \frac{-\sinh 2\alpha \pm i}{\cosh 2\alpha} \quad , \quad x\delta = -\frac{1}{8} \quad \text{and} \quad x \rightarrow 0 \quad (5.9)$$

Inserting these values into (5.2)-(5.6), one can easily check that the solution given in Eq.s (4.13)-(4.17) is recovered and the α -dependence drops out. Notice, in particular, that even if the vanishing value for x renders both f_- and f_+ trivial, the function $X(r)$ and the scalars η_a do not become constant since the products $x\beta$ and $x\delta$ are non zero. On the other hand, the fact that $x\beta$ is purely imaginary, makes all hyperbolic functions become periodic.

The structure of the general solution (5.2)-(5.6) clearly indicates that different choices of the integration constants do not necessarily yield a classical geometry with a pathological behavior. However, when one regards the supergravity as the low-energy description of string theory, one should ask which of all possible choices in the classical context have some physical interpretation from the string viewpoint. A first very natural possibility is to consider an orbifold compactification with an internal volume $V \neq V_c$ (recall that at $V_c = \alpha'^2/4$ the non-BPS D-particle becomes “extremal”, in some sense). In fact, provided that $V > V_c$, the D-particle remains a stable configuration [8], even if it does not exhibit any more the Fermi-Bose degeneracy which, at first order, was responsible for the no-force condition. However, since this condition does not hold at higher orders, it is not necessary to focus on the critical volume any more, and one can hope that the departure from extremality will eventually lead to develop an event horizon which, hiding all singularities in a casual disconnected region from the physical space, would make the solution (5.2)-(5.6) free of singularities.

Another possibility could be to consider a stable bound state obtained by displacing along the transverse directions the stack of D-branes in a sphere of radius $r \sim Q^{1/3}$. This kind of mechanism for resolving the repulsive singularities which we have found in our solution, is similar to the one advocated in [25], where these problems have been discussed in great detail. In our non-BPS situation, this option, however, deserves further investigation. Of course, one could also imagine more exotic possibilities that rely on very different settings, with different source terms and more bulk fields that couple to them. These configurations would need a different truncation of the original (1,1) supergravity theory discussed in Section 3, and are clearly not accomplished by the solution (5.2)-(5.6).

The highly non-trivial role that stable non-BPS D-branes could play in deeper understanding of non-perturbative dualities in string theory and eventually on non-supersymmetric versions of the AdS/CFT correspondence, clearly makes quite challenging to find some positive answers to these open problems.

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Appendix A

In this appendix we present the diagrammatic calculations of the leading and next-to-leading terms in the large distance expansion of the fields emitted by a non-BPS D-particle. To do this, we first rewrite the bulk part of the action (4.1) in terms of canonical normalized fields (see Eq. (3.17)) and get

$$S_{\text{bulk}} = \int d^6x \sqrt{-\det G} \left[\frac{1}{2\kappa_{orb}^2} \mathcal{R}(G) - \frac{1}{2} \partial_\mu \hat{\varphi} \partial^\mu \hat{\varphi} - \frac{1}{2} \partial_\mu \hat{\eta}_a \partial^\mu \hat{\eta}_a - \frac{1}{4} e^{\kappa_{orb} \hat{\varphi}} \hat{F}^2 \right] \quad (\text{A.1})$$

with $G_{\mu\nu} = \eta_{\mu\nu} + 2\kappa_{orb} \hat{h}_{\mu\nu}$. Expanding (A.1) in κ_{orb} , we get

$$S_{\text{bulk}} = S_0 + \kappa_{orb} S_I + \mathcal{O}(\kappa_{orb}^2) \quad (\text{A.2})$$

where S_0 is the free action and

$$S_I = S_{\varphi\varphi h} + S_{AAh} + S_{\eta\eta h} + S_{hhh} \quad . \quad (\text{A.3})$$

The four terms in S_I describe respectively the interaction of a graviton with two dilatons, of a graviton with two gauge fields, of a graviton with two scalars, and the coupling among three gravitons.

The interactions of the bulk fields with the D-brane are encoded in the boundary part of the action (4.1), which, at the linearized level, is

$$S_{\text{boundary}} = \int d^6x \left(J^{\mu\nu} \hat{h}_{\mu\nu} + J \hat{\varphi} + J_a \hat{\eta}_a + J^\mu \hat{A}_\mu \right) \quad (\text{A.4})$$

where the currents are

$$\begin{aligned} J_{\mu\nu}(x) &= \kappa_{orb} M \eta_{\mu 0} \eta_{\nu 0} \delta^5(\vec{x}) \quad , \quad J(x) = J_a(x) = \frac{\kappa_{orb} M}{2} \delta^5(\vec{x}) \quad , \\ J_\mu(x) &= -\sqrt{2} \kappa_{orb} M \eta_{\mu 0} \delta^5(\vec{x}) \quad . \end{aligned} \quad (\text{A.5})$$

The quadratic (and higher order) terms of the boundary action will not be needed in our calculations since they give rise only to tadpole diagrams which vanish in dimensional regularization.

In this theory, the one-point function of a generic bulk field $\widehat{\Psi}(x)$ is given by

$$\int [D\widehat{h} D\widehat{\varphi} D\widehat{A} D\widehat{\eta}] \widehat{\Psi}(x) e^{iS_0} e^{iS_I} e^{iS_{\text{boundary}}} \equiv \langle \widehat{\Psi}(x) e^{iS_I} e^{iS_{\text{boundary}}} \rangle . \quad (\text{A.6})$$

Expanding the two exponentials, we generate a perturbative series whose various terms correspond to diagrams that contain a different number of bulk and boundary interactions. The first two diagrams in this series are represented in Figure 3 and describe, respectively, the leading and next-to-leading terms in the large distance expansion of the classical bulk fields. In particular, the leading contribution is obtained from (A.6) by neglecting S_I and expanding at first order the exponential containing S_{boundary} . Applying this procedure to the dilaton, we find that the leading contribution to its one-point function is

$$\widehat{\varphi}^{(1)}(x) = i \int d^6 y \langle \widehat{\varphi}(x) J(y) \widehat{\varphi}(y) \rangle = \frac{\kappa_{orb} M}{2} \int \frac{d^5 k}{(2\pi)^5} \frac{e^{ik \cdot x}}{k^2} . \quad (\text{A.7})$$

By using the following expression for the Fourier transform

$$\int \frac{d^d k}{(2\pi)^d} \frac{e^{ik \cdot x}}{k^{2\alpha}} = \frac{2\alpha}{2^{2\alpha} \pi^{d/2}} \frac{\Gamma\left(\frac{d}{2} + 1 - \alpha\right)}{\Gamma(1 + \alpha)} \frac{1}{(d - 2\alpha)} \frac{1}{|x|^{d-2\alpha}} , \quad (\text{A.8})$$

we easily see that (A.7) becomes

$$\widehat{\varphi}^{(1)}(x) = \frac{1}{\kappa_{orb}} \frac{1}{4} \frac{Q}{r^3} \quad (\text{A.9})$$

where Q is the parameter defined in Eq. (4.12). This is precisely the leading term at large distance of the dilaton produced by the non-BPS D-particle (see Eq. (4.7)). In a similar manner we can compute the asymptotic behavior of the other bulk fields and find complete agreement with the results reported in Eqs (4.8)-(4.11). We now compute the next-to-leading order of the one-point function (A.6). This is obtained by expanding the exponential of S_I at first order and the exponential of S_{boundary} at second order. Applying this procedure to the dilaton, we find two contributions corresponding to the diagrams in Figure 5, so that we can write

$$\widehat{\varphi}^{(2)}(x) = A_{\varphi}^{\varphi AA}(x) + A_{\varphi}^{\varphi hh}(x) . \quad (\text{A.10})$$

The first term, due to the coupling of a dilaton with two gravitons (see Figure 5a), is equal to

$$\begin{aligned} A_{\varphi}^{\varphi hh}(x) &= i^3 \int d^6 y \int d^6 z \int d^6 u \\ &\quad \left\langle \widehat{\varphi}(x) \widehat{\varphi}(y) J(y) \partial_{\mu} \widehat{\varphi}(z) \partial_{\nu} \widehat{\varphi}(z) \left(\widehat{h}^{\mu\nu}(z) - \frac{1}{2} \widehat{h}_{\tau}^{\tau}(z) \eta^{\mu\nu} \right) \widehat{h}_{\rho\sigma}(u) J^{\rho\sigma}(u) \right\rangle . \end{aligned} \quad (\text{A.11})$$

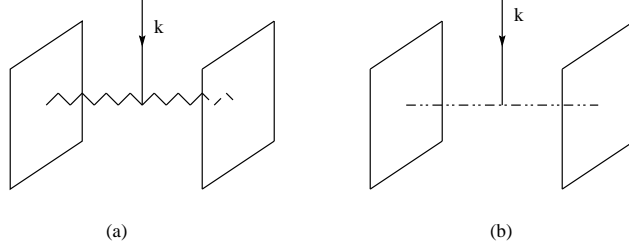


Figure 5: The next to leading order contributions for the dilaton. Diagram (a) represents the two boundary contribution via graviton exchange while diagram (b) corresponds to the gauge field contribution.

By performing all contractions and using the explicit expressions for the propagators, it is not difficult to see that $A_\varphi^{\varphi hh}(x) = 0$. Moreover, it is interesting to notice that this result holds in any space-time dimension. The second term in (A.10) corresponds to the diagram of Figure 5b that is given by

$$\begin{aligned} A_\varphi^{\varphi AA}(x) &= \frac{i\kappa_{orb}}{8} \int d^6 y \int d^6 z \int d^6 u \langle \hat{\varphi}(x) \hat{\varphi}(y) \hat{F}^2(y) \hat{A}_\mu(z) J^\mu(z) \hat{A}_\nu(u) J^\nu(u) \rangle \\ &= \kappa_{orb}^3 M^2 \int \frac{d^5 k}{(2\pi)^5} \frac{e^{ik \cdot x}}{k^2} \int \frac{d^5 p}{(2\pi)^5} \frac{p \cdot (p+k)}{p^2 (k+p)^2} . \end{aligned} \quad (\text{A.12})$$

The second integral in (A.12) can be easily evaluated with standard techniques. For the sake of generality we give the result of the previous integral for an arbitrary value d of the number of transverse directions. One gets

$$\int \frac{d^d p}{(2\pi)^d} \frac{p \cdot (p+k)}{k^2 p^2 (k+p)^2} = -\frac{1}{2} \frac{1}{(4\pi)^{d/2}} B\left(\frac{d}{2}-1, \frac{d}{2}-1\right) \Gamma\left(2-\frac{d}{2}\right) (k^2)^{d/2-2} . \quad (\text{A.13})$$

Inserting this result in (A.12) and using (A.8) for $\alpha = -1/2$ and $d = 5$, we finally get

$$\hat{\varphi}^{(2)}(x) = -\frac{1}{\kappa_{orb}} \frac{1}{8} \left(\frac{Q}{r^3}\right)^2 \quad (\text{A.14})$$

which agrees with the next-to-leading term of Eq. (4.7).

The same calculations that lead to $A_\varphi^{\varphi hh}(x) = 0$, also imply that $A_\eta^{\eta hh}(x) = 0$. In fact, the gravitational couplings of the scalar fields $\hat{\eta}_a$ are the same as those of the dilaton, and therefore also this diagram does not contribute. On the other hand, since $\hat{\eta}_a$ does not couple to any other bulk field, from the vanishing of $A_\eta^{\eta hh}(x)$ we can deduce that $\hat{\eta}_a$ does not receive any correction at the next-to-leading order. Actually, also the higher orders for these fields are vanishing and thus the leading term for $r \rightarrow \infty$ already gives the exact result (see Eq. (4.14)).

With this same method we can compute the next-to-leading term for the gauge field \hat{A}_0 . This is the sum of two terms which arise from the bulk interaction of two gauge fields with a dilaton and a graviton respectively, namely

$$\hat{A}_0^{(2)}(x) = A_A^{A\varphi A}(x) + A_A^{AhA}(x) \quad . \quad (\text{A.15})$$

Finally, the next-to-leading term of the graviton is produced by the bulk interactions involving three gravitons, two dilatons and one graviton, two gauge fields and one graviton, and two scalars and one graviton, that is

$$\hat{h}_{\mu\nu}^{(2)}(x) = A_{\mu\nu}^{hhh}(x) + A_{\mu\nu}^{h\varphi\varphi}(x) + A_{\mu\nu}^{hAA}(x) + A_{\mu\nu}^{h\eta\eta}(x) \quad . \quad (\text{A.16})$$

All terms in (A.15) and (A.16) can be computed following the procedure outlined before, and after some lengthy algebra one gets precisely the next-to-leading behavior of the gauge field and the metric written in Eq.s (4.9)-(4.11).

Appendix B

In this appendix we explicitly derive the non-BPS D-particle solution (4.13)-(4.17), and the most general one presented in Eq.s (5.2)-(5.6). For the sake of generality we start from a D -dimensional action containing the metric, the dilaton, the scalars η_a and a $(p+1)$ -form R-R potential with $p < D-3$. The case of the non-BPS D-particle, considered in Section 2, can be obtained by taking in all our equations $p=0$ and $D=6$. However, our equations can also be used to derive the non-BPS solution in $D=10$ discussed in Section 2. Actually they can also be used for the usual BPS D-branes in ten dimensions. We start from the following action

$$S = S_{\text{bulk}} + S_{\text{boundary}} \quad (\text{B.1})$$

where

$$S_{\text{bulk}} = \frac{1}{2\kappa_{orb}^2} \int d^D x \sqrt{-\det G} \left[\mathcal{R}(G) - \partial_\mu \varphi \partial^\mu \varphi - \partial_\mu \eta_a \partial^\mu \eta_a - \frac{1}{2(p+2)!} e^{a\varphi} F_{p+2}^2 \right] \quad (\text{B.2})$$

and

$$S_{\text{boundary}} = -M \int d^{p+1} \xi e^{-\frac{a}{2}\varphi - \frac{1}{2}\sum_a \eta_a} \sqrt{-\det G_{\alpha\beta}} + M \int A_{p+1} \quad (\text{B.3})$$

where κ_{orb} has been defined in (3.12), while $M = NM_p$ with

$$M_p = \frac{\sqrt{2} T_p}{(2\pi)^2 \kappa_{orb} V^{1/2}} \quad . \quad (\text{B.4})$$

As mentioned above, we treat simultaneously the cases of non-BPS branes in both $D = 6$ and $D = 10$. This can be done by taking the constant a to be given by

$$a = \frac{D - 4 - 2p}{\sqrt{D - 2}} . \quad (\text{B.5})$$

Clearly, if $D = 10$ in (B.4) we have to put κ_{10} in place of κ_{orb} and delete the factor of $(2\pi)^2 V^{1/2}$. Moreover, in the ten dimensional case there are no scalars η_a and in the case of the non-BPS branes there is no R-R field. Finally for the BPS branes there is no factor of $\sqrt{2}$ in the brane tension (B.4).

By varying the action (B.1), we get the equations of motion for the various fields. In particular, we have

$$\frac{1}{\sqrt{-\det G}} \partial_\mu \left(\sqrt{-\det G} G^{\mu\nu} \partial_\nu \varphi \right) - \frac{a}{4} e^{a\varphi} \frac{1}{(p+2)!} F_{p+2}^2 = \frac{a}{2} T(x) \delta^d(\vec{x}) \quad (\text{B.6})$$

for the dilaton,

$$\frac{1}{\sqrt{-\det G}} \partial_\mu \left(\sqrt{-\det G} G^{\mu\nu} \partial_\nu \eta_a \right) = \frac{1}{2} T(x) \delta^d(x) \quad (\text{B.7})$$

for the scalars and

$$\partial_{\mu_1} \left(\sqrt{-\det G} e^{a\varphi} G^{\mu_1 \nu_1} \dots G^{\mu_{p+2} \nu_{p+2}} \frac{F_{\nu_1 \dots \nu_{p+2}}}{(p+1)!} \right) = -2 M \kappa_{orb}^2 G_0^{\mu_2} \dots G_p^{\mu_{p+2}} \delta^d(\vec{x}) \quad (\text{B.8})$$

for the R-R field. Finally, the Einstein equations for the metric can be written in a simple form by first evaluating their trace, and then plugging it back into the original equations, obtaining

$$\begin{aligned} \mathcal{R}_{\mu\nu} - \frac{e^{a\varphi}}{2(p+2)!} \left[(p+2) F_{\mu\mu_2 \dots \mu_{p+2}} F_{\nu}^{\mu_2 \dots \mu_{p+2}} - G_{\mu\nu} \frac{(1+p)}{4} F_{p+2}^2 \right] \\ - \partial_\mu \varphi \partial_\nu \varphi - \partial_\mu \eta_a \partial_\nu \eta_a = T(x) \left(G_{\mu\alpha} G_{\nu\beta} G^{\alpha\beta} - \frac{p+1}{4} G_{\mu\nu} \right) \delta^d(\vec{x}) \end{aligned} \quad (\text{B.9})$$

where

$$T(x) = -M \kappa_{orb}^2 e^{-\frac{a}{2}\varphi - \frac{1}{2}\sum_a \eta_a} \frac{\sqrt{-\det G_{\alpha\beta}}}{\sqrt{-\det G}} . \quad (\text{B.10})$$

We now solve the previous equations using the following *Ansatz* for the metric

$$ds^2 = B^2(r) \eta_{\alpha\beta} dx^\alpha dx^\beta + F^2(r) \delta_{ij} dx^i dx^j \quad (\text{B.11})$$

where $\alpha, \beta = 0, \dots, p$ and $i, j = p+1, \dots, d \equiv D-p-1$, and assuming that all other fields are functions only of the radial coordinate r . Under these assumptions, the

dilaton equation (B.6) becomes

$$\begin{aligned} \frac{1}{r^{d-1}} \left(r^{d-1} B^{p+1} F^{d-2} \varphi' \right)' + \frac{a}{4} e^{a\varphi} F^{d-2} B^{-p-1} \left(A'_{01\dots p} \right)^2 \\ = \frac{a}{2} B^{p+1} F^d T(x) \delta^d(\vec{x}) \quad , \end{aligned} \quad (\text{B.12})$$

the scalar equation (B.7) becomes

$$\frac{1}{r^{d-1}} \left(r^{d-1} B^{p+1} F^{d-2} \eta'_a \right)' = \frac{1}{2} B^{p+1} F^d T(x) \delta^d(\vec{x}) \quad , \quad (\text{B.13})$$

while R-R field equation (B.8) becomes

$$\frac{1}{r^{d-1}} \left(r^{d-1} B^{-p-1} F^{d-2} e^{a\varphi} A'_{01\dots p} \right)' = 2 M \kappa_{orb}^2 \delta^d(\vec{x}) \quad (\text{B.14})$$

where $' \equiv d/dr$. Finally, from the Einstein equations (B.10) we get

$$\begin{aligned} F^{-2} \left\{ -\xi'' - (\log F)'' - \frac{d-1}{r} (\log F)' - (p+1) \left[(\log B)' \right]^2 + \xi' (\log F)' \right. \\ \left. - (d-2) \left[(\log F)' \right]^2 \right\} - F^{-2} (\varphi')^2 - F^{-2} \sum_a (\eta'_a)^2 \\ - \frac{e^{a\varphi}}{2} \frac{d-2}{D-2} F^{-2} B^{-2(p+1)} \left(A'_{01\dots p} \right)^2 = -\frac{p+1}{D-2} T(x) \delta^d(\vec{x}) \end{aligned} \quad (\text{B.15})$$

for the components R_r^r ,

$$\begin{aligned} F^{-2} \left\{ -(\log B)'' - (\log B)' \left[\xi' + \frac{d-1}{r} \right] \right\} \\ - \frac{e^{a\varphi}}{2} \left(-\frac{d-2}{D-2} \right) F^{-2} B^{-2(p+1)} \left(A'_{01\dots p} \right)^2 = \frac{d-2}{D-2} T(x) \delta^d(\vec{x}) \end{aligned} \quad (\text{B.16})$$

for the components R_α^α , and

$$\begin{aligned} F^{-2} \left[-(\log F)'' - \frac{d-1}{r} (\log F)' - (\log F)' \xi' - \frac{\xi'}{r} \right] \\ - \frac{e^{a\varphi}}{2} \frac{p+1}{D-2} B^{-2p-2} \left(A'_{01\dots p} \right)^2 = -\frac{p+1}{D-2} T(x) \delta^d(\vec{x}) \end{aligned} \quad (\text{B.17})$$

for the components $R_{\bar{\alpha}}^{\bar{\alpha}}$ where the index $\bar{\alpha}$ corresponds to the angular variables. In these equations we have introduced the function

$$\xi = (p+1) \log B + (d-2) \log F \quad . \quad (\text{B.18})$$

Following for example [34], we now multiply Eq. (B.16) by a factor of $(p+1)$ and Eq. (B.17) by a factor of $(d-2)$, and then sum the two expressions. In this way we see that the function ξ obeys a simple differential equation, namely

$$\left[r^{2d-3} (e^\xi)' \right]' = 0 \quad . \quad (\text{B.19})$$

This is the Laplace equation in $2d-1$ dimensions and its most general solution can be written as

$$e^\xi = \hat{C} + C \left(\frac{Q_p}{r^{d-2}} \right)^2 \quad (\text{B.20})$$

where \hat{C} and C are arbitrary constants, and, for later convenience, we have introduced the dimensionful quantity

$$Q_p = \frac{2 \kappa_{orb}^2 M_p}{(d-2) \Omega_{d-1}} \quad . \quad (\text{B.21})$$

In order to have an asymptotically flat metric, we must choose $\hat{C} = 1$, and thus we can write

$$e^\xi \equiv B^{p+1} F^{d-2} = f_-(r) f_+(r) \quad (\text{B.22})$$

with

$$f_\pm(r) = 1 \pm x \frac{Q_p}{r^{d-2}} \quad ; \quad x^2 = -C \quad . \quad (\text{B.23})$$

Inserting Eq. (B.22) into Eq.s (B.12)-(B.17), we get

$$\frac{e^{-\xi}}{r^{d-1}} \left(r^{d-1} e^\xi \varphi' \right)' + \frac{a}{4} e^{a\varphi} B^{-2(p+1)} \left(A'_{01\dots p} \right)^2 = \frac{a}{2} F^2 T(x) \delta^d(\vec{x}) \quad (\text{B.24})$$

for the dilaton,

$$\frac{e^{-\xi}}{r^{d-1}} \left(r^{d-1} e^\xi \eta'_a \right)' = \frac{1}{2} F^2 T(x) \delta^d(\vec{x}) \quad (\text{B.25})$$

for the scalars η_a ,

$$\frac{1}{r^{d-1}} \left(r^{d-1} e^{a\varphi} B^{-(p+1)} F^{d-2} A'_{01\dots p} \right)' = 2 M \kappa_{orb}^2 \delta^d(\vec{x}) \quad (\text{B.26})$$

for the R-R field, while Eq. (B.16) can be rewritten as

$$\begin{aligned} \frac{e^{-\xi}}{r^{d-1}} \left[r^{d-1} e^\xi (\log B)' \right]' - \frac{e^{a\varphi}}{2} \frac{d-2}{D-2} B^{-2(1+p)} \left(A'_{01\dots p} \right)^2 \\ = -\frac{d-2}{D-2} F^2 T(x) \delta^d(\vec{x}) \quad . \end{aligned} \quad (\text{B.27})$$

Multiplying Eq. (B.24) by $2(d-2)/(D-2)$ and Eq. (B.27) by a , and then summing the resulting expressions we get:

$$\left(r^{d-1} e^\xi Y'\right)' = 0 \quad (\text{B.28})$$

where

$$Y \equiv \frac{D-2}{d-2} \log B + \frac{2}{a} \varphi \quad . \quad (\text{B.29})$$

The solution of this equation is

$$e^Y \equiv B^{(D-2)/(d-2)} e^{2\varphi/a} = \left(\frac{f_-(r)}{f_+(r)}\right)^\epsilon \quad (\text{B.30})$$

where ϵ is an arbitrary integration constant⁸. Actually the most general solution of Eq. (B.28) admits an additional arbitrary constant which, however, we have fixed by requiring that Y vanish for $r \rightarrow \infty$. Using Eq. (B.17) in Eq. (B.15), and expressing B and F in terms of ξ, Y and φ we can rewrite Eq. (B.15) as follows

$$\begin{aligned} \frac{e^{a\varphi}}{2} B^{-2(p+1)} (A'_{01\dots p})^2 &= \frac{4}{D-2} \left[\frac{a^2(D-2)}{4} + (p+1)(d-2) \right] \left(\frac{\varphi'}{a} \right)^2 + \\ &+ \sum_a (\eta'_a)^2 + \xi'' - \frac{1}{d-2} (\xi')^2 - \frac{\xi'}{r} + \frac{(d-2)(p+1)}{D-2} \left[(Y')^2 - Y' \left(\frac{4\varphi'}{a} \right) \right] \quad . \end{aligned} \quad (\text{B.31})$$

Let us start by examining the case of a non-BPS D-brane in $D = 6$. When we use Eq. (B.31) in the dilaton equation (B.24), the latter becomes

$$\begin{aligned} \frac{e^{-\xi-2\varphi/(1-p)+(d-2)(p+1)Y/2}}{r^{d-1}} \left[r^{d-1} e^{\xi-(d-2)(p+1)Y/2} \left(e^{\frac{2}{1-p}\varphi} \right)' \right]' + \sum_a (\eta'_a)^2 \\ - \frac{d-1}{d-2} (\xi')^2 - 2(d-1) \frac{\xi'}{r} + \frac{(d-2)(p+1)}{4} (Y')^2 = F^2 T(x) \delta^{(5-p)}(\vec{x}) \quad . \end{aligned} \quad (\text{B.32})$$

Using Eq.s (B.22) and (B.30) with $D = 6$ for the functions ξ and Y , and the fact that the scalar fields η_a satisfy the same homogeneous equation as Y and therefore are given by

$$e^{\eta_a} = \left(\frac{f_-(r)}{f_+(r)} \right)^\delta \quad , \quad (\text{B.33})$$

⁸Here and in the following, when $a = 0$ (e.g. $p = 1$ in $D = 6$ and $p = 3$ in $D = 10$) the equations are ill-defined. However, their general solutions are valid also in these cases.

one can see that the dilaton equation (B.32) becomes

$$\begin{aligned} & \frac{e^{-\xi-2\varphi/(1-p)+(d-2)(p+1)Y/2}}{r^{d-1}} \left[r^{d-1} e^{\xi-(d-2)(p+1)Y/2} \left(e^{\frac{2}{1-p}\varphi} \right)' \right]' + C \frac{d-2}{r^2} \left(\frac{Q_p}{r^{d-2}} \right)^2 e^{-2\xi} \\ & \times \left[-16(d-2)\delta^2 + 4(d-1) - (d-2)^2(p+1)\epsilon^2 \right] = F^2 T(x) \delta^{(5-p)}(\vec{x}) . \end{aligned} \quad (\text{B.34})$$

This must be considered together with the equation (B.26) for the R-R field, which becomes

$$\frac{1}{r^{d-1}} \left(r^{d-1} e^{\xi-(p+1)(3-p)Y/2+\frac{4}{1-p}\varphi} A'_{01\dots p} \right) = 2 M \kappa_{orb}^2 \delta^{(5-p)}(\vec{x}) . \quad (\text{B.35})$$

Finally, using Eq.s (B.29) and (B.28), Eq. (B.27) can be rewritten as follows

$$\frac{e^{-\xi}}{r^{d-1}} \left[r^{d-1} e^{\xi} \left(\frac{2\varphi'}{1-p} \right)' \right] + \frac{1}{2} e^{4\varphi/(1-p)-(p+1)(d-2)Y/2} \left(A'_{01\dots p} \right)^2 = -T(x) \delta^{(5-p)}(\vec{x}) . \quad (\text{B.36})$$

In the following we want to find the most general solution of Eq.s (B.31) and (B.34)-(B.36) excluding the origin where the boundary action is located and corresponding to vanishing values of φ and $A_{01\dots p}$ for $r \rightarrow \infty$. Under these conditions, we find that (B.34) is solved by

$$e^{2\varphi/(1-p)} = \left(\frac{\cosh X(r) + \gamma \sinh X(r)}{\cosh \alpha + \gamma \sinh \alpha} \right) \left(\frac{f_-(r)}{f_+(r)} \right)^{(d-2)(1+p)\epsilon/4} \quad (\text{B.37})$$

where

$$X(r) = \alpha + \beta \log \frac{f_-(r)}{f_+(r)} , \quad (\text{B.38})$$

provided that the following relation is satisfied

$$\begin{aligned} & -4(d-2)(x\beta)^2 - 16(d-2)(x\delta)^2 + 4(d-1)x^2 \\ & + (d-2)^2(p+1)(x\epsilon)^2 \left(\frac{(d-2)(p+1)}{4} - 1 \right) = 0 . \end{aligned} \quad (\text{B.39})$$

Inserting the solution for the dilaton into Eq. (B.35) and neglecting again the source term, we find that the R-R field is given by

$$A_{01\dots p} = \sqrt{2(\gamma^2 - 1)} \left[\frac{\sinh X(r)(\cosh \alpha + \gamma \sinh \alpha)}{\cosh X(r) + \gamma \sinh X(r)} - \sinh \alpha \right] \quad (\text{B.40})$$

where the overall constant has been determined in terms of γ through Eq. (B.36).

Finally from Eq.s (B.22), (B.30) and (B.37), one can find the explicit expressions for the components of the metric:

$$B^2 = \left(\frac{\cosh X(r) + \gamma \sinh X(r)}{\cosh \alpha + \gamma \sinh \alpha} \right)^{-(d-2)/2} \left(\frac{f_-(r)}{f_+(r)} \right)^{\epsilon(d-2)/2[1-(d-2)(p+1)/4]} \quad (\text{B.41})$$

and

$$F^2 = \left(\frac{\cosh X(r) + \gamma \sinh X(r)}{\cosh \alpha + \gamma \sinh \alpha} \right)^{(p+1)/2} (f_-(r)f_+(r))^{\frac{2}{(d-2)}} \times \left(\frac{f_-(r)}{f_+(r)} \right)^{-\epsilon(p+1)/2[1-(d-2)(p+1)/4]} . \quad (\text{B.42})$$

Eq.s (B.33), (B.37) and (B.40)-(B.42) represent the most general solution of the field equations derived from the action (B.2) which describe a static, spherically symmetric configuration, with asymptotically flat geometry and vanishing v.e.v.'s at infinity for the gauge and scalar fields. The solution depends on five arbitrary parameters α , β , γ , δ and x (ϵ is in fact determined in terms of the others through Eq. (B.39)). Setting $p = 0$ one recovers the solution presented in Section 5 (see Eq.s (5.2)-(5.6)).

In writing this solution, we have not used the precise form of the source terms, or equivalently we have not imposed that the behavior of the various fields at infinity be consistent with what follows from the boundary state, which is given by

$$\varphi \simeq \frac{1-p}{4} \frac{Q_p}{r^{d-2}} \quad , \quad \eta_a \simeq \frac{1}{4} \frac{Q_p}{r^{d-2}} \quad , \quad A_{01\dots p} \simeq -\frac{Q_p}{r^{d-2}} \quad (\text{B.43})$$

and

$$G_{\alpha\beta} \simeq \eta_{\alpha\beta} \left(1 - \frac{d-2}{4} \frac{Q_p}{r^{d-2}} \right) \quad , \quad G_{ij} \simeq \delta_{ij} \left(1 + \frac{p+1}{4} \frac{Q_p}{r^{d-2}} \right) . \quad (\text{B.44})$$

If we impose that our general solution behaves for large r as required by the previous conditions, we must choose the integration constants as follows

$$x\beta = \pm \frac{i}{4} \quad , \quad \gamma = \frac{-\sinh 2\alpha \pm i}{\cosh 2\alpha} \quad , \quad x\delta = -\frac{1}{8} \quad \text{and} \quad x \rightarrow 0 \quad , \quad (\text{B.45})$$

with α arbitrary. For this choice, it is not difficult to see that the α dependence drops out and the solution is given by

$$\eta_a = \frac{1}{4} \frac{Q_p}{r^{3-p}} \quad (\text{B.46})$$

$$e^{2\varphi} = \left(1 + \sin \frac{Q_p}{r^{3-p}} \right)^{\frac{1-p}{2}} \quad (\text{B.47})$$

$$F^2 = \left(1 + \sin \frac{Q_p}{r^{3-p}}\right)^{\frac{1+p}{4}} \quad (\text{B.48})$$

$$B^2 = \left(1 + \sin \frac{Q_p}{r^{3-p}}\right)^{\frac{p-3}{4}} \quad (\text{B.49})$$

$$A_{01\dots p} = -1 + \frac{\cos \frac{Q_p}{r^{3-p}}}{1 + \sin \frac{Q_p}{r^{3-p}}} . \quad (\text{B.50})$$

For $p = 0$ this is precisely the solution (4.13)-(4.17).

In the final part of this appendix we use the general equations we have derived to find the solution corresponding to the non-BPS branes in $D = 10$ discussed in Section 2. In this case we have to switch off the scalar fields η_a and the R-R field. Keeping this in mind, the dilaton equation (B.24) becomes

$$\frac{e^{-\xi}}{r^{d-1}} \left(r^{d-1} e^{\xi} \varphi' \right)' = \frac{a}{2} F^2 T(x) \delta^{(9-p)}(x) , \quad (\text{B.51})$$

whereas the metric equations (B.27) and (B.17) become respectively

$$\frac{e^{-\xi}}{r^{d-1}} \left[r^{d-1} e^{\xi} (\log B)' \right]' = -\frac{7-p}{8} F^2 T(x) \delta^{(9-p)}(x) \quad (\text{B.52})$$

and

$$\frac{e^{-\xi}}{r^{d-1}} \left[r^{d-1} e^{\xi} (\log F)' \right]' + \frac{\xi'}{r} = \frac{p+1}{8} F^2 T(x) \delta^{(9-p)}(x) . \quad (\text{B.53})$$

Finally, Eq. (B.31) becomes

$$-\xi'' + \frac{(\xi')^2}{d-2} + \frac{\xi'}{r} - \frac{(p+1)(d-2)}{8} \left[(Y')^2 - 4 Y' \frac{\varphi'}{a} \right] = 2 \left(\frac{2\varphi'}{a} \right)^2 . \quad (\text{B.54})$$

If we use Eq.s (B.18) and (B.29), we can easily see that they coincide with (2.10), provided that the boundary term is omitted.

Neglecting for a moment the origin, where the boundary term is located, the most general solution of Eq.s (B.51) and (B.52) is given by

$$e^{\varphi} = \left(\frac{f_-(r)}{f_+(r)} \right)^{\nu} , \quad B^2 = \left(\frac{f_-(r)}{f_+(r)} \right)^{\lambda} \quad (\text{B.55})$$

where λ and ν are constants to be determined and f_- and f_+ are given in Eq. (B.23) with the substitution of Q_p with \hat{Q}_p (the ten dimensional non-BPS D-brane charge defined in Eq. (2.6)). From the previous equations and Eq. (B.30) we get

$$\epsilon = \frac{4}{7-p} \lambda + \frac{2}{a} \nu . \quad (\text{B.56})$$

Inserting in Eq. (B.54) the Eq.s (B.22), (B.30) and the first equation in (B.55) one gets

$$8 \left(\frac{8-p}{7-p} \right) - (p+1)(7-p) \left(\epsilon - 2 \frac{\nu}{a} \right)^2 = 8 \nu^2 \quad . \quad (\text{B.57})$$

Finally imposing that the solution matches also the boundary term we get

$$x = \frac{p-3}{8\sqrt{2}\nu} = \frac{7-p}{16\lambda} \quad . \quad (\text{B.58})$$

Eq.s (B.57) and (B.58) imply that

$$\epsilon = 0 \quad , \quad x = \sqrt{\frac{7-p}{8(8-p)}} \quad , \quad \lambda = \frac{7-p}{16x} \quad , \quad \nu = \frac{p-3}{8\sqrt{2}x} \quad . \quad (\text{B.59})$$

Taking into account that the kinetic term for the dilaton in Eq.s (2.4) and (B.2) have a factor 2 of difference in the normalization we see that Eq.s (B.59) reproduce the solution for the ten dimensional non-BPS D-branes given in Eq.s (2.13) and (2.14).

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